1.1 Angles

Line. Let A and B be two distinct points. We can draw a unique line passing through A and B, and we will call it *line* AB. By this, we mean a set of points that stretches from infinity on one side, passes through A, then through B, and goes on to infinity on the other side.

Ray. If we drop the part of the line AB that lies "before" the point A, what remains is called *ray* AB. Thus, ray AB is the portion of the line AB that starts at A, continues through B, and on past B to infinity. Here, point A is called the *endpoint* of ray AB.

We will now use the concept of a ray to define the notion of angle:

Angle. An *angle* is formed by rotating a ray around its endpoint. The ray in its initial position is called the *initial side* of the angle, while the ray in its location after the rotation is the *terminal side* of the angle. The endpoint of the ray is the *vertex* of the angle.

Note the close relation between the notions of *angle* and *rotation* in the definition above. There is more to an angle than just a vertex and two rays with endpoints located in the vertex. Given a vertex, an initial side and a terminal side, we have not defined an angle in a unique way: there are obviously two different angles sharing the same initial and terminal sides — one by rotating a ray from the initial side to the terminal side clockwise, and one by performing a rotation from the initial side to the terminal side counterclockwise.[1] Therefore, the key word in the definition of an angle is *rotation*.

Positive and Negative Angles. As we identified (in a way) an angle with a rotation, and as we distinguish two kinds of rotations: clockwise and counterclockwise, we can use this property to classify angles into two categories accordingly. If an angle is formed by a counterclockwise rotation, we will say that the angle is *positive*. If an angle is formed by a clockwise rotation, the angle will be said to be *negative*.

Degree Measure. The measure of angles allows us to compare angles that do not share the same vertex and initial side. One of the most common units for measuring angles is the *degree*. It is defined by assigning a numerical value of $360°$ to the angle that corresponds to a (counterclockwise) full rotation. Alternatively, it could be said that an angle of $360°$ is the smallest positive angle such that its initial and terminal sides coincide.

Right Angle, Straight Angle. Having chosen a measure for an angle corresponding to a full rotation, we can find measures for angles that are fractions thereof:

[1] In fact, there are infinitely many angles for given initial and terminal sides — see the paragraph on coterminal angles below.

An angle obtained by rotating a ray by a half of a full rotation has a measure of $\frac{1}{2} \times 360^\circ = 180^\circ$, and is called a *straight angle.*

An angle of 90° corresponds to a quarter of a full rotation, and is called a *right angle.*

Acute Angles, Obtuse Angles. Angles measuring between 0° and 90° are called *acute angles.* Angles measuring between 90° and 180° are called *obtuse angles.*

Complementary and Supplementary Angles. Two positive angles, the sum of which is 90° are called *complementary.* If the sum of two positive angles is 180°, they are said to be *supplementary.*

Note that if two angles are complementary, they are necessarily both acute. If two angles are supplementary, they are either both right, or one is acute and the other obtuse.

Standard Position. For reference purposes, it is convenient to introduce coordinate axes x and y. An angle is said to be in *standard position* if its vertex is located at the origin, and its initial side is along the positive x-axis.

An angle in standard position is said to lie in the quadrant in which its terminal side lies. An acute angle lies in quadrant I, and an obtuse angle lies in quadrant II.

In standard position, a right angle has its terminal side along the positive y-axis, while a straight angle has its terminal side along the negative x-axis. These two are examples of *quadrantal angles*—angles that have their terminal side along any coordinate axis.[2]

Coterminal Angles. So far, we have only seen angles corresponding to a full rotation or less, but there is nothing preventing the rotation from going on after a full rotation: in that way, we get angles measuring more than 360°.

Obviously, the terminal side of any angle with a measure greater than 360° coincides with the terminal side of some angle less than 360°. This is the case with any two angles differing by a full rotation or multiple thereof. Such angles are called *coterminal angles.*

[2] It could be said that these are angles that lie in between quadrants, or which delineate quadrants.