1.2 Angle Relationships and Similar Triangles

Last time, we defined an angle in terms of a rotation, and assigned a sign to it depending on the sense of the rotation (clockwise or counterclockwise). This *dynamic* aspect of an angle should be regarded as fundamental. Today, however, we will not be concerned with rotations, but we will look at angles that appear in some simple (*static*) geometric pictures, such as two intersecting lines or a triangle. In these situations, we do not care about which side is the initial and which is the terminal, and we will only be concerned with the absolute value of the angle's measure. Angles will therefore be marked by an arc without arrow.

Vertical Angles. Let us take an angle and extend its sides to form another angle. The pair of angles thus obtained is called a pair of *vertical angles*. Two intersecting lines always make two pairs of vertical angles.

An important property of vertical angles is that *vertical angles have equal measures.*[1]

Parallel and Transversal Lines. Two lines are called *parallel* if they lie in the same plane and do not intersect. A (third) line that intersects two parallel lines is called a *transversal.*

A picture with two parallel lines and a transversal shows plenty (eight) angles. It is interesting to identify angles with equal measures. For this, we make use of the property of vertical angles above, and the property that *corresponding angles have equal measures.* (How many angles with different measures are there?)

Angle Sum of a Triangle. Using what we have learned so far, we can actually prove a very important property of any triangle: that *the sum of the measures of the angles of a triangle is* $180°$.[2] To show this, all it takes is to extend two sides of a triangle, draw a line parallel to the third side through the common vertex of the other two, identify vertical angles and parallel and transversal lines, and apply the two properties about measures of vertical and corresponding angles.

Types of Triangles. We classify triangles according to angles and sides:

Angles

Acute triangle: all angles acute;

[1] We can easily convince ourselves of this if we recall the dynamic definition of an angle, and visualize the rotation of one line (the "initial sides") until it reaches the other line (the "terminal sides"). Obviously, both angles were produced by the same rotation, and therefore correspond to the same fraction of a full rotation.

[2] Although it is very important to remember this property, the actual proof that we will see is not included in the book, and students are not required to learn it. It is presented just as a demonstration of application of the previous two properties.

Right triangle: one right angle;

Obtuse triangle: one obtuse angle.

Sides

Equilateral triangle: all sides equal;

Isosceles triangle: two sides equal;

Scalene triangle: no sides equal.

Congruent Triangles. Two triangles are called *congruent* if they have the same shape and size. It is worthwile noting that the triangles need not be oriented the same way to be congruent. Two triangles are congruent if one can be lined up with the other by performing one of these transformations: slide, rotation or reflection, or any combination of these three transformations. Corresponding sides of congruent triangles have the same length, and the corresponding angles have the same measure.

Similar Triangles. Two triangles are said to be *similar* if they have the same shape, but not necessarily the same size. Again, the triangles can be oriented in different ways, or even be a mirror reflection of each other. As with congruent triangles, corresponding angles must have equal measures, but in this case, corresponding sides are not equal but proportional.